GENERALIZED SPECTRAL DOMAIN ANALYSIS OF PLANAR STRUCTURES HAVING SEMI-INFINITE GROUND PLANES

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Accurate, efficient techniques that utilize the general Galerkin's method in Fourier transform domain are formulated to compute the quasi-TEM parameters of planar structures having semi-infinite strips. Examples include coplanar waveguides with and without the conductor backing and microstrips with a parallel slot in the ground plane. Computed results for typical cases of symmetrical, nonsymmetrical, single and multiple strip coplanar waveguide and microstrip-slot structures are presented.

INTRODUCTION

This paper deals with the implementation of the general Galerkin's method in spectral domain for the computation of the quasi-TEM parameters of a class of planar structures that have coplanar semi-infinite ground planes. Examples of such structures include single and multiple strip coplanar waveguides (CPW) with or without the conductor backing and microstrips with a parallel slot in the ground plane as shown in Figure 1. For the case of the coplanar waveguides, even though the spectral domain full wave analysis has been conducted [1,2], the quasi-TEM analysis of such structures has been confined either to approximate conformal mapping [3,4] or to finite difference and other methods for shielded structures [5,6]. For the microstrip-slot structures, the shielded structure has been analyzed for its quasi-TEM parameters by using the spectral domain technique [7] and the frequency-dependent parameters of the open structure have been computed by using the method of lines [8,9]. The Galerkin's method in Fourier transform domain which has been employed for the accurate, efficient computation of the quasi-TEM parameters of many planar structures has not been used for the structures shown in Figure 1 primarily because the static Green's function for these structures leads to the Fredholm integral equation of the first kind with a Cauchy-type singular kernel for which satisfactory solution techniques have not been developed [10]. In this paper it is shown that this equation can be solved by using the general Galerkin method leading to accurate efficient computation of the quasi-TEM parameters of these structures. The computed results are presented for typical cases and compared with other results wherever a meaningful comparison can be made.

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THE SPECTRAL DOMAIN GREEN'S FUNCTIONS

The spectral domain Green's functions interrelating the charges, potentials, and fields at the surfaces must be set up in terms of variables that are compatible with the implementation of the general Galerkin method. Due to the semi-infinite extensions of the ground strips, the integral equations in space domain or the corresponding algebraic equations in the Fourier transform domain are set up such that the unknown variable that is to be expanded in terms of convenient basis functions is defined over a finite interval in space domain. For the structures shown in Figure 1, the defining Green's functions in spectral domain are set up as:

$$\tilde{\rho}(\alpha,d) = \tilde{Y}(\alpha)\tilde{E}_x(\alpha,d) \quad (1)$$

for the CPW's, and

$$\tilde{\phi}_1(\alpha,d) = \tilde{h}_{11}(\alpha)\tilde{\rho}_1(\alpha,d) + \tilde{h}_{12}(\alpha)\tilde{E}_{x2}(0,\alpha)$$

$$\tilde{\rho}_2(\alpha,0) = \tilde{h}_{21}(\alpha)\tilde{\rho}_1(\alpha,d) + \tilde{h}_{22}(\alpha)\tilde{E}_{x2}(0,\alpha)\ , \quad (2)$$

for the microstrip-slot structure.

For the case of the CPW's, the tangential electric field in the slots is used as a "source" parameter and for the microstrip-slot structures a hybrid form formulation is used since two different "source" parameters coexist, i.e., distributed charges on the upper surface and the tangential electric field on the lower surface are used as a source function. These Green's functions are derived in a straightforward manner by writing the solution of the transformed Poisson's equation with the given boundary conditions and noting that $\tilde{E}_x(\alpha) = j\alpha\,\tilde{\phi}(\alpha)$, and are found to be:

$$\tilde{Y}(\alpha) = -j\varepsilon_o \mathrm{sgn}\alpha \frac{(1+\varepsilon_r^2)+2\varepsilon_r\coth|\alpha|h}{1+\varepsilon_r\coth|\alpha|h} \quad (3)$$

for CPW of Figure 1a without conductor backing, and

$$\tilde{Y}(\alpha) = -j\varepsilon_o \mathrm{sgn}\alpha(1+\varepsilon_r\coth|\alpha|h) \quad (4)$$

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for CPW of Figure 1b with conductor backing, and

$$\tilde{h}_{11}(\alpha) = \frac{1}{\varepsilon_o|\alpha|(1+\varepsilon_r\coth|\alpha|h)} \quad (5a)$$

$$\tilde{h}_{12}(\alpha) = -j\varepsilon_r \frac{1}{\alpha(\sinh|\alpha|h+\varepsilon_r\cosh|\alpha|h)} \quad (5b)$$

$$\tilde{h}_{21}(\alpha) = \frac{-\varepsilon_r}{(\sinh|\alpha|h+\varepsilon_r\cosh|\alpha|h)} \quad (5c)$$

$$\tilde{h}_{22}(\alpha) = -j\varepsilon_o \mathrm{sgn}\alpha \frac{(1+\varepsilon_r^2)+2\varepsilon_r\coth|\alpha|h}{1+\varepsilon_r\coth|\alpha|h} \quad (5d)$$

where $\mathrm{sgn}\alpha = \begin{cases} +1, & \alpha > 0 \\ -1, & \alpha < 0 \end{cases}$.

SOLUTION METHOD

Because of the odd symmetry of $\tilde{Y}(\alpha)$ for both cases of the CPW's and $\tilde{h}_{22}(\alpha)$ which results in a Cauchy-type singular kernel in space domain, it is seen that different spaces for electric field function set and charge function set must be defined in order to solve the associated integral equation by utilizing the local basis approximation method (LBAM). We have used the LBAM, which is not without its shortcomings, simply because it is readily adaptable to nonsymmetrical and multiple line problems. In order to illustrate the solution method we consider the simplest case of a single strip coplanar waveguide without the conductor backing. By exploring the relationships between charges, potential and electric field, the appropriate test function set found for charges when the basis function set for E_x is gate function (piecewise constant), is the same gate function set but defined on the intersecting segments as shown in Figure 2 for a single line case. That is, E_x is expanded in terms of gate functions as,

$$\tilde{E}_x(x) = \sum_{i=1}^{N} a_i \pi(x-x_i) \text{ or } \tilde{E}_z(\alpha) = \sum_{i=1}^{N} a_i e^{jx_i d}\tilde{\pi}(\alpha) \quad (6)$$

and the test function for charges,

$$f_i(x) = \Pi(x-x_i-b/2) \text{ or } \tilde{f}_i(\alpha) = e^{j(x_i+b/2)\alpha}\tilde{\Pi}(\alpha), \quad (7)$$

where

$$\tilde{\Pi}(\alpha) = \frac{2}{b\alpha}\sin\frac{b\alpha}{2} \text{ and } \Pi(x) \triangleq \begin{cases} 1/b, & \text{for } -b/2<x<b/2 \\ 0, & \text{otherwise} \end{cases}$$

Substitution of (6) in (1) and applying Parseval's theorem to the left hand side after taking the inner product with the test function set of (7) leads to (for the discretization scheme of Figure 2a):

$$\sum_{j=1}^{N} K_{ij}a_j = 0, \ i=1,2,..,n_1-1,n_1+m_1+1,..,N-1 \quad (8)$$

Potential across the slots,

$$V = \sum_{j=1}^{n_1} a_j = -\sum_{j=n_1+m_1+1}^{N} a_j$$

where

$$K_{ij} = 1/(2\pi)\int_{-\infty}^{\infty}\tilde{Y}(\alpha)\tilde{\Pi}^2(\alpha)e^{-j(x_i-x_j+b/2)\alpha}d\alpha \quad (9)$$

In order to speed up the convergence process we have decomposed $\tilde{Y}(\alpha)$ in a homogeneous part and the remainder as in [11] leading to:

$$K_{ij} = -\frac{\varepsilon_o(1+\varepsilon_r)}{\pi b}\{D_{n+1}-2D_n+D_{n-1}\}+\varepsilon_o\varepsilon_r(1-\varepsilon_r)/\pi \int_0^{\infty}\frac{(1-\coth|\alpha|h)}{1+\varepsilon_r\coth|\alpha|h}\tilde{\Pi}^2(\alpha)\sin(n+1/2)\alpha\, d\alpha \quad (10)$$

with $n \triangleq i-j$ and $D_n \triangleq (n+1/2)\log|(n+1/2)b|$.

Now, K_{ij} is easily evaluated leading to the solution of Eq. (8) for unknown coefficients a_j's, and then the charged induced on the strip is determined by

$$Q = \sum_{j=1}^{N}\left(\sum_{i=n_1}^{n_1+m_1} K_{ij}\right)a_j. \quad (11)$$

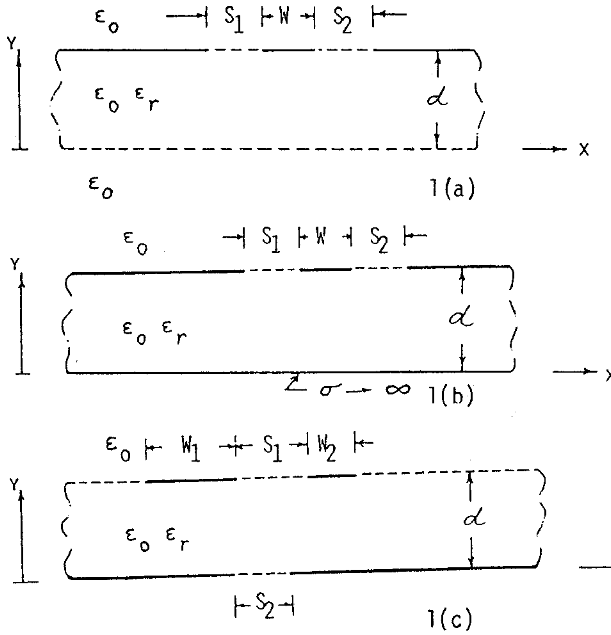
It should be noted that the two possibilities shown in Figure 2 for discretization lead to a lower bound and an upper bound solution for charge Q, and that the formulation scheme is readily generalized to cover the multiple coupled coplanar waveguide lines. The formulations for the CPW with a conductor backing and microstrip-slot structures are similar to the one presented above.

RESULTS

The quasi-TEM parameters (effective dielectric constants and impedances) of several typical structures have been computed on a CDC 6400 computer and a few examples are shown in Figures 3 through 6. These examples were chosen to illustrate the versatility and the accuracy of the techniques formulated in the paper. The computation time varies from case to case but is generally in the range of 0.5 ~ 5 seconds for each computation shown in the figures.

REFERENCES

1. J.B. Knorr, K.D. Kuchler; IEEE Trans. MTT, V. 23, 1975, pp. 541-548.
2. Y.C. Shih, T. Itoh; Electron Lett., V. 18, 1982, pp. 538-540.
3. C.P. Wen; IEEE Trans. MTT, V. 17, 1969, pp. 1087-1090.
4. M.E. Davies et al.; IEEE Trans. MTT, V. 21, 1973, pp. 594-596.
5. T. Hatsuda; IEEE Trans. MTT, V. 23, 1975, pp. 795-802.
6. D.A. Rowe, B.Y. Lao; IEEE Trans. MTT, V. 31, 1983, pp. 911-915.
7. T. Itoh, A.S. Hebert; IEEE Trans. MTT, V. 26, 1978, pp. 820-826.
8. R.K. Hoffmann, J. Siegl; IEEE Trans. MTT, V. 30, 1982, pp. 1211-1215.
9. Ü. Schulz, R. Pregla; Arch. Elek. Übertragung, 1980, pp. 169-173.
10. K.E. Atkinson; SIAM J. Num. Anal., V. 9, 1972, pp. 284-299.
11. H. Lee, V.K. Tripathi; IEEE MTT-S Microwave Symp. Dig., 1983, pp. 571-573.



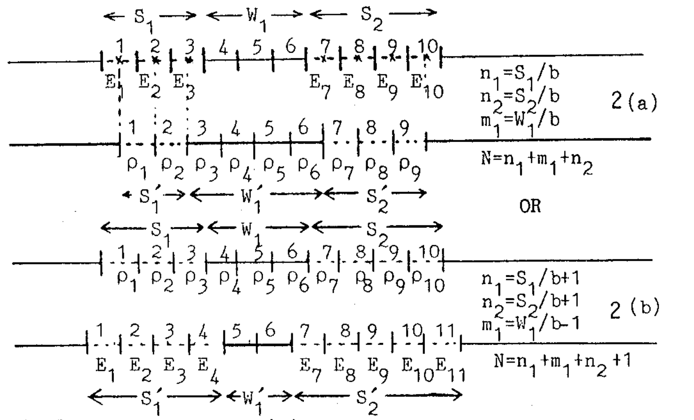


Fig.1. Schematics of (a) Coplanar Waveguide (b) CPW with a conductor backing (c) Coupled microstrip slot structure.

Fig.2. Discretization schemes for electric field and charges in a single strip CPW.

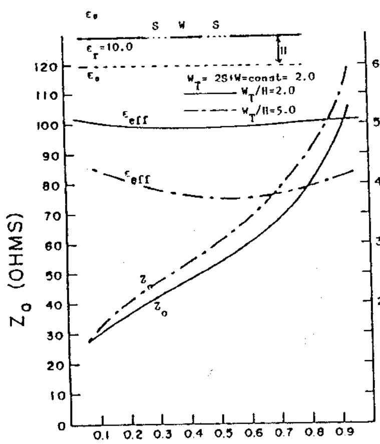


Fig.3

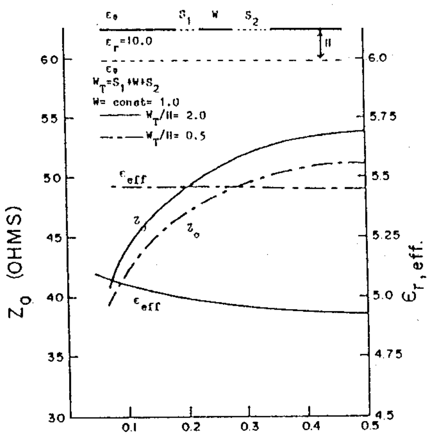


Fig.4

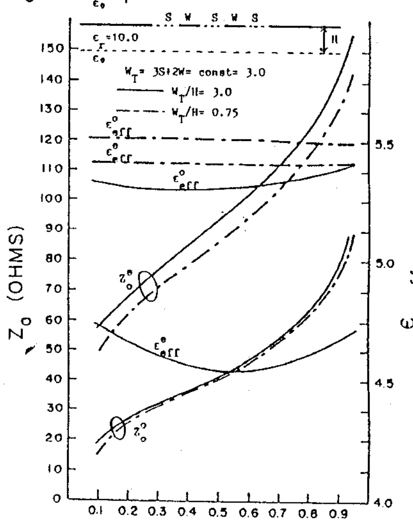


Fig.5

Fig.3.Symmetric Coplanar Waveguide Parameters

Fig.3.Asymmetric Coplanar Waveguide parameters

Fig.5.Nonsymmetrical Coupled CPW normal mode parameters.

Fig.6.(a)Deviations of microstrip parameters as a function of slot width W/h=1.0, ϵ_r=9.8 , - - -from (8) for λ=100h (b)Coupled microstrips normal mode parameters vs S_2

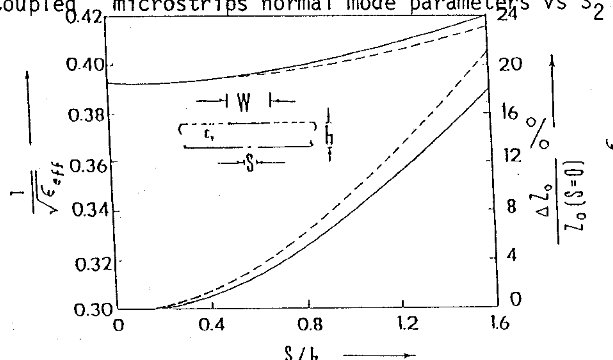


Fig.6(a)

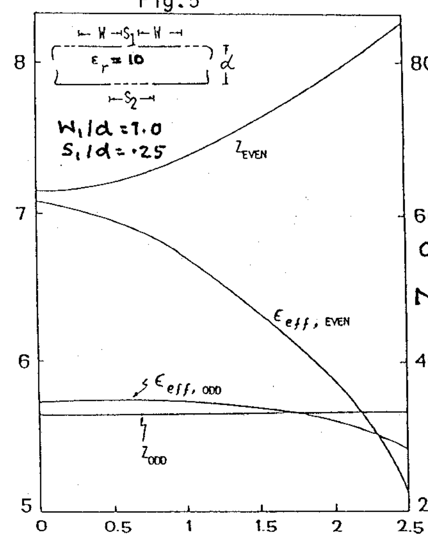


Fig.6b SLOT WIDTH, S_2/d